Complexity of Robinson tiling

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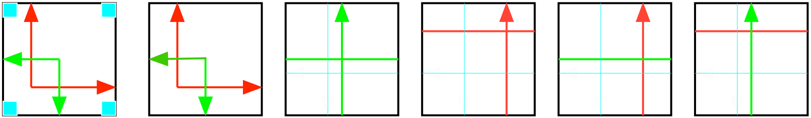
Table of Contents

Table of Contents

Robinson tileset

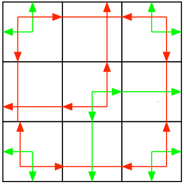

(a) (b) (c) (d) (e) (f)

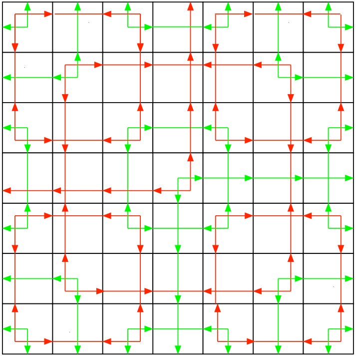
- Tiles of type (a) are called *bumpy corners*
- Of type (b) are called *corners*
- All the other are called *arms*.

Tiling

- Tiling: covering of the plane by interior disjoint tiles.
- Aperiodic tiling: no invariance by translation
- All Robinson tilings are aperiodic!

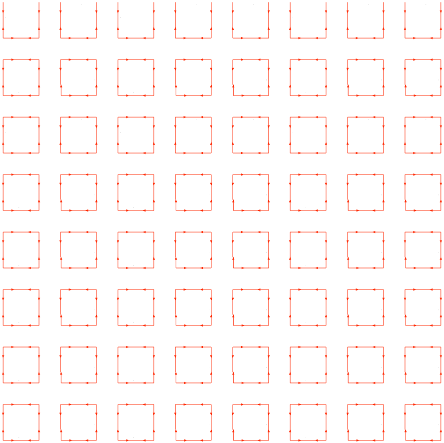
Supertiles



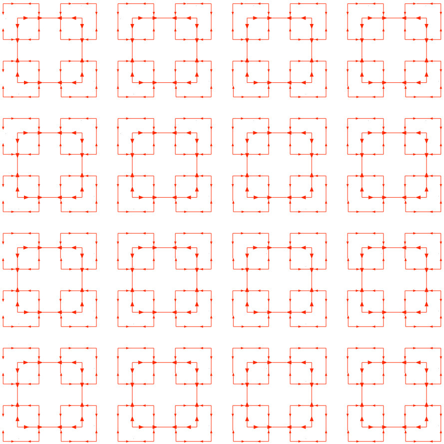


Supertiles of second and third rank.

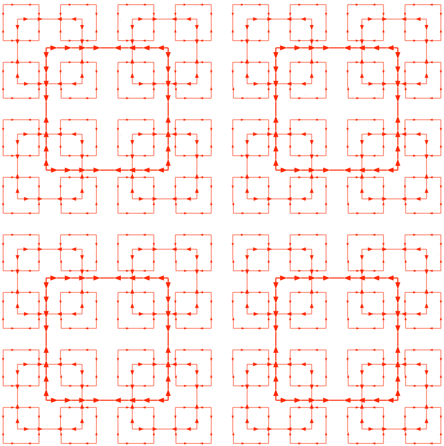
Hierarchy



Hierarchy



Hierarchy



Hierarchy

The Robinson tiling can be either made of

- one infinite order supertile
- contain two or four infinite order supertiles

The question: how many distinct blocks of size $n \times n$ there are in Robinson tiling made of one infinite order supertile?

Table of Contents

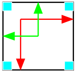
Motivation

What is local self-assembly?

- units of the growing cluster must be added one by one
- decisions are local, i.e. according to tiles within a fixed distance
- no infomation must be stored between the steps

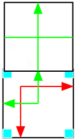
Motivation is to prove that it is impossible to assemble a Robinson tiling just by chance!

Deceptions



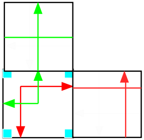
- Deceptions: adding tiles one by one may lead to a pattern that cannot be further extended.

Deceptions



- Deceptions: adding tiles one by one (*self-assembly*) may lead to a pattern that cannot be further extended.

Deceptions



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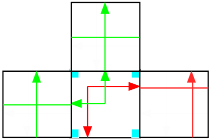

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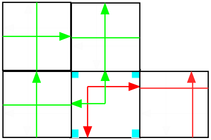

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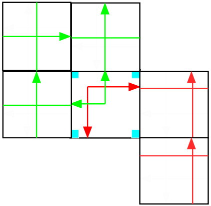

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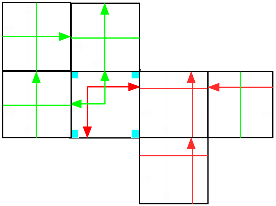
Deceptions



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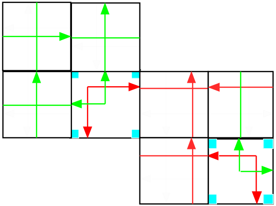
Deceptions



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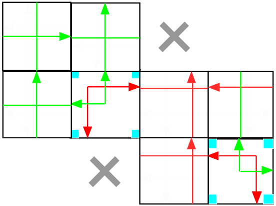

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Alternative representation

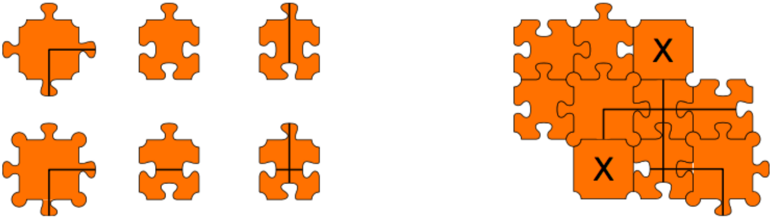


Table of Contents

Notation

Denote $n \times n$ square by S_n. Mark cells of S_n with a symbol

- C if we have chosen this cell to be a corner
- V if it is *possible* to place a corner tile
- black dot ($\bullet$) if otherwise

Step 1

C	•	C	•	C
•	V	•	V	•
C	•	C	•	C
•	V	•	V	•
C	•	C	•	C

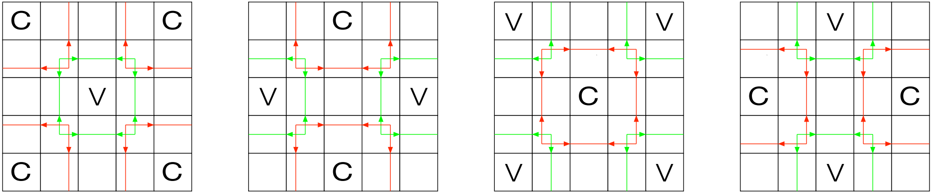
V	•	V	•	V
•	C	•	C	•
V	•	V	•	V
•	C	•	C	•
V	•	V	•	V

•	C	•	C	•
V	•	V	•	V
•	C	•	C	•
V	•	V	•	V
•	C	•	C	•

•	V	•	V	•
C	•	C	•	C
•	V	•	V	•
C	•	C	•	C
•	V	•	V	•

Four variants for corner tiles.

Step 2



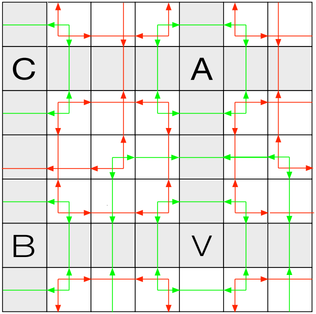
Four variants to place a corner tile when there is already a corner tile in [2,2].

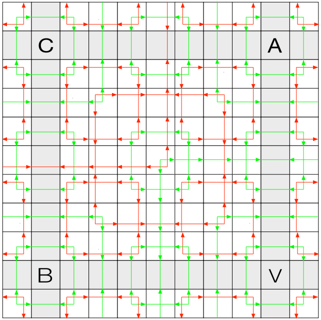
Lemma

Lemma

If two squares have the same number of vacant places, then there is the same number of possibilities to complete both to a correct pattern.

Bijection





- All the cells are defined except for two rows and two columns.
- One of 'crossroads' have to be a corner tile

Counting the vacant places

If $n = 2k$ then the number of vacant places after the first step is $k \times k$.

If $n = 2k + 1$ then:

- corner tile in position [1,1] : $k \times k$
- corner tile in position [2,2] : $(k + 1) \times (k + 1)$
- corner tile in position [1,2] : $k \times (k + 1)$
- corner tile in position [2,1] : $k \times (k + 1)$

Notation

Denote by

- A_1 – number of all correct tilings of S_2 with a corner tile in [1,1]
- A_n – number of all correct tilings of S_n
- B_n – number of all correct tilings of S_{2n+1} with a corner tile in [1,2]

Recurrence relations

$$
\begin{aligned}
A_{2n} &= 4 \cdot A_n \\
A_{2n+1} &= A_n + A_{n+1} + 2 \cdot B_n \\
B_{2n} &= 2 \cdot A_n + 2 \cdot B_n \\
B_{2n+1} &= 2 \cdot A_{n+1} + 2 \cdot B_n
\end{aligned}
$$

Values of A_1 and B_1 can be found by an exhaustive search:

$$
\begin{aligned}
A_1 &= 56; \\
B_1 &= 124.
\end{aligned}
$$

Guess the solution

The solution for recurrence relations can be written as

$$A_n = a(n) \cdot A_1 + b(n) \cdot B_1,$$

where

$$a(n) = 5n^2 - 12n \cdot 2^{\lfloor \log_2 n \rfloor} + 8 \cdot 2^{2\lfloor \log_2 n \rfloor} \quad (1)$$
$$b(n) = -2n^2 + 6n \cdot 2^{\lfloor \log_2 n \rfloor} - 4 \cdot 2^{2\lfloor \log_2 n \rfloor} \quad (2)$$

The sum of (1) and (2) gives us:

Voila

Theorem

For any Robinson tiling made of one infinite order supertile, once $n > 1$, the number of distinct $n \times n$ square blocks is given by

$$A(n) = 32n^2 + 72n \cdot 2^{\lfloor \log_2 n \rfloor} - 48 \cdot 2^{2\lfloor \log_2 n \rfloor}.$$

Conjecture

For any seed the probability to construct $k \times k$ square patch of Robinson tiling via local self-assembly process tends to 0.

Thank you for your attention!