Complexity of Robinson tiling
Abstract

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Raphael Robinson in his work on undecidability of the domino problem [1] introduced the set of six tiles depicted in Figure 1. The tiles can be rotated and reflected, one tile can fit to the other only in such a way that the arrow head matches arrow tail and each 2×2 block must contain exactly one *bumpy corner*, the leftmost in Figure 1.

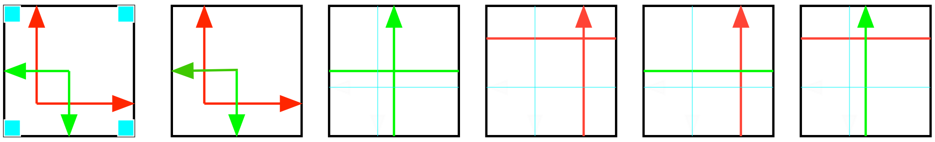

(a) (b) (c) (d) (e) (f)

Figure 1: Tiles of type (a) are called *bumpy corners*, tiles of type (b) are called *corners*, all the other tiles are called *arms*.

It is possible to tile the Euclidean plane with copies of this six tiles, but only in an *aperiodic* way. The key to this result is that any Robinson tiling has a hierarchical structure (see Figure 2). For all n, it is possible to define the *supertiles* of rank n inductively. Bumpy corner tiles are said to be supertiles of the first rank, supertiles of second and third rank are shown in Figure 2. An increasing union of supertiles of rank n is called an infinite order supertile. The Robinson tiling can be either made of only *one* infinite order supertile or contain *two* or *four* infinite order supertiles.

We will prove that the number of distinct blocks of size $n \times n$ (with $n \geq 2$) that could appear in a Robinson tiling made of one infinite order supertile is defined by the formula

$$A(n) = 32n^2 + 72n \cdot 2^{\lfloor \log_2 n \rfloor} - 48 \cdot 2^{2\lfloor \log_2 n \rfloor}.$$

Similar method for counting the number of factors in the *paper folding sequence* was used by Jean-Paul Allouche in [2].

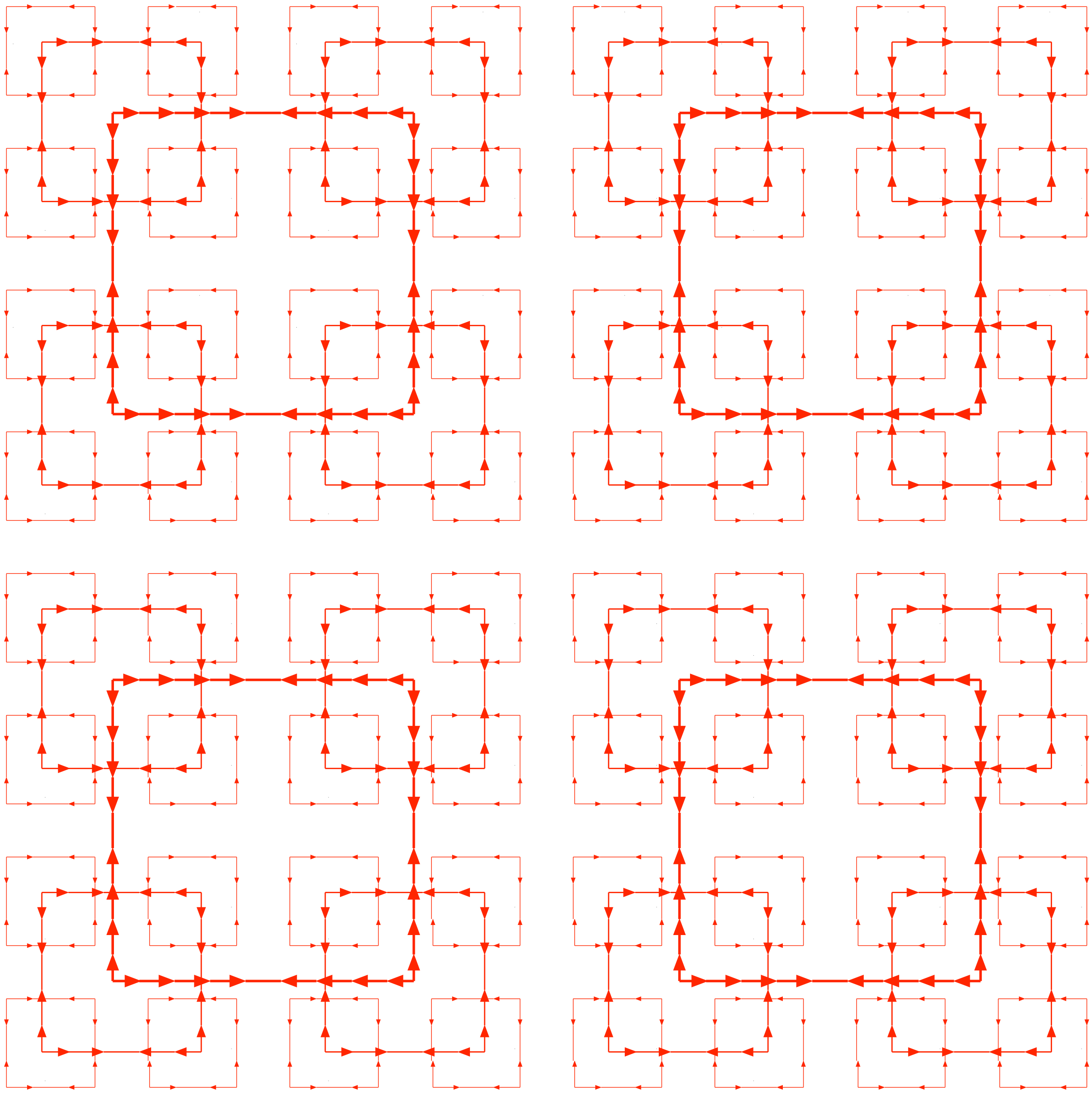
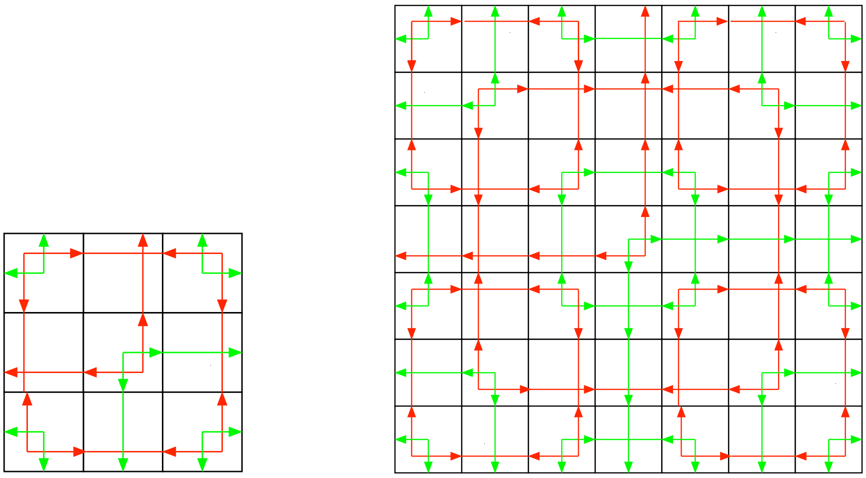


Figure 2: Hierarchy.



(a) (b)

Figure 3: Supertiles of second and third rank.

References

[1] R. Robinson, *Undecidability and nonperiodicity for tilings of the plane*, Invent. Math. **12** (1971) 177–209.

[2] J.-P. Allouche, *The number of factors in a paperfolding sequence*, Bull. Austral. Math. Soc. **46**, 23–32 (1992).