Complexity of Robinson tiling

Abstract

Galanov Ilya
galanov@lipn.univ-paris13.fr

Raphael Robinson in his work on undecidability of the domino problem \[1\] introduced the set of six tiles depicted in Figure 1. The tiles can be rotated and reflected, one tile can fit to the other only in such a way that the arrow head matches arrow tail and each \(2 \times 2\) block must contain exactly one bumpy corner, the leftmost in Figure 1.

![Figure 1: Tiles of type (a) are called bumpy corners, tiles of type (b) are called corners, all the other tiles are called arms.](image)

It is possible to tile the Euclidean plane with copies of this six tiles, but only in an aperiodic way. The key to this result is that any Robinson tiling has a hierarchical structure (see Figure 2). For all \(n\), it is possible to define the supertiles of rank \(n\) inductively. Bumpy corner tiles are said to be supertiles of the first rank, supertiles of second and third rank are shown in Figure 2. An increasing union of supertiles of rank \(n\) is called an infinite order supertile. The Robinson tiling can be either made of only one infinite order supertile or contain two or four infinite order supertiles.

We will prove that the number of distinct blocks of size \(n \times n\) (with \(n \geq 2\)) that could appear in a Robinson tiling made of one infinite order supertile is defined by the formula

\[ A(n) = 32n^2 + 72n \cdot 2^{ \lfloor \log_2 n \rfloor } - 48 \cdot 2^{2\lfloor \log_2 n \rfloor}. \]

Similar method for counting the number of factors in the paper folding sequence was used by Jean-Paul Allouche in \[2\].
Figure 2: Hierarchy.
Figure 3: Supertiles of second and third rank.

References
